

ANALYSIS OF STEEL REINFORCED FUNCTIONALLY GRADED CONCRETE BEAM CROSS SECTIONS



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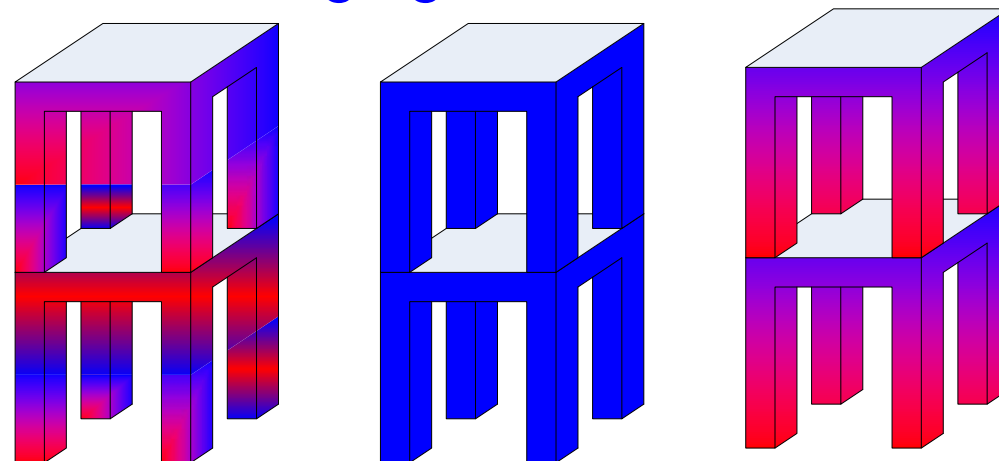
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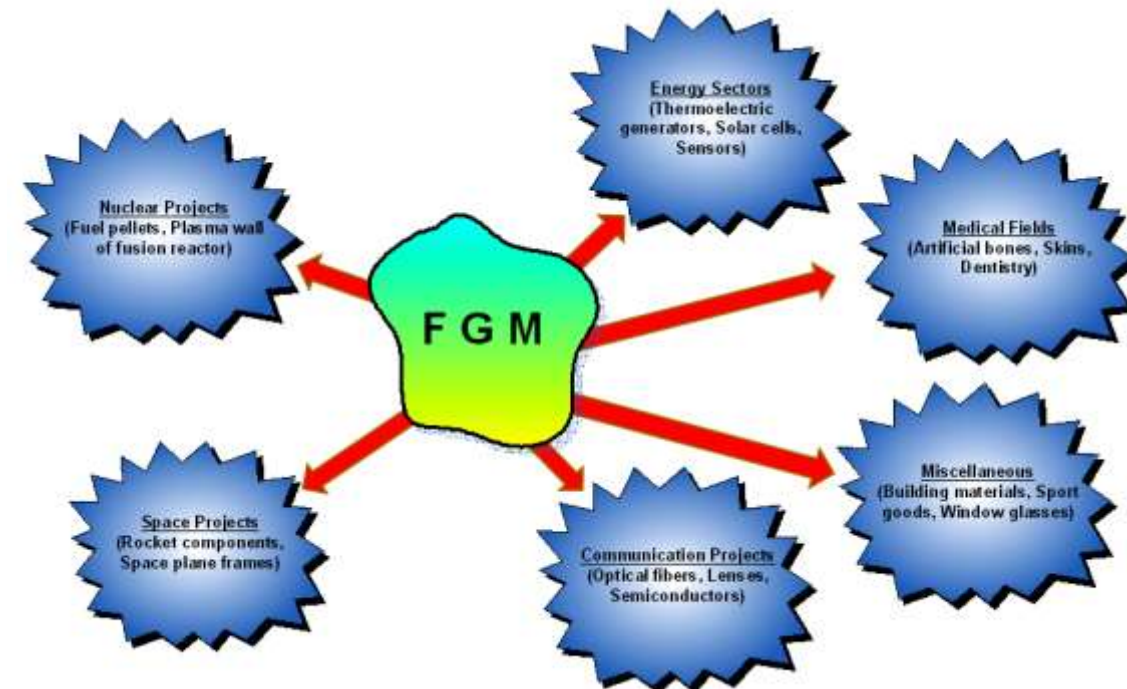
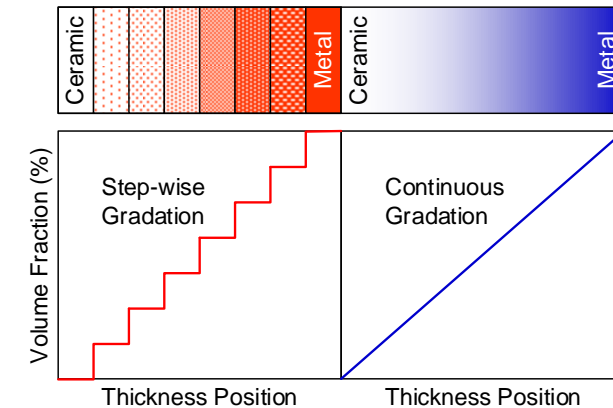
INTRODUCTION

- Concrete is designed and manufactured with **homogeneous** properties.
- The homogeneity is effective to ensure the **safety** of a structure.
- **On the contrary** to the homogeneity assumed in the analyses and design, steel Reinforced Concrete (RC) structure elements in built structures are mostly found as **graded concrete material**.
- The non-homogeneous material property is a result of **mixing, placing, and curing procedures**, in addition to the **segregation and accumulation** of the **aggregates** during the mixing.



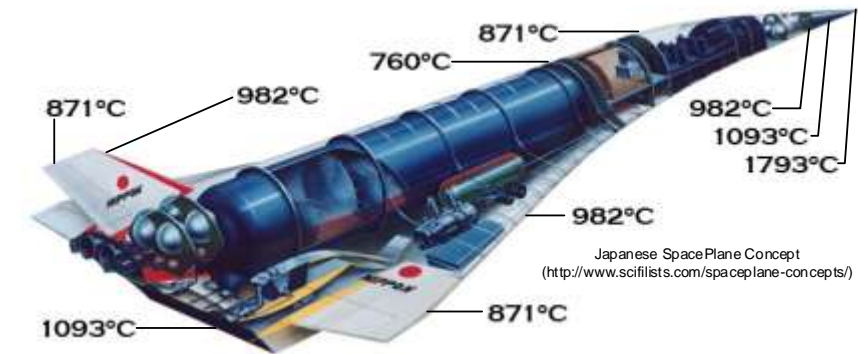
FGM

- Functionally Graded Material (FGM) is a made by combining **two or more materials** where the essential **properties** are **varied** over **a specified orientation** to obtain some desired function abilities.
- In FGM compositions, two or more material properties are **blended functionally** to improve material performances.



FGM

- Functionally Graded Material (FGM)
- FGM = Functionally Graded Material
- Initiated by Japanese scientists in 1984 (Koizumi)
- Formed by a **varying percentage of constituents** in any desired spatial direction
- Results in **specific physical and mechanical properties**



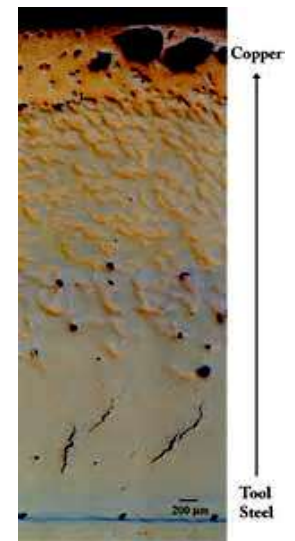
Bones



Ceramic-steel dental implant



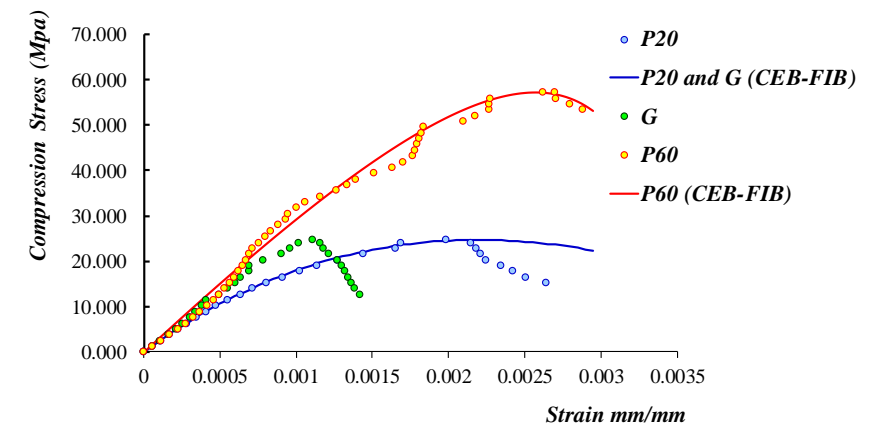
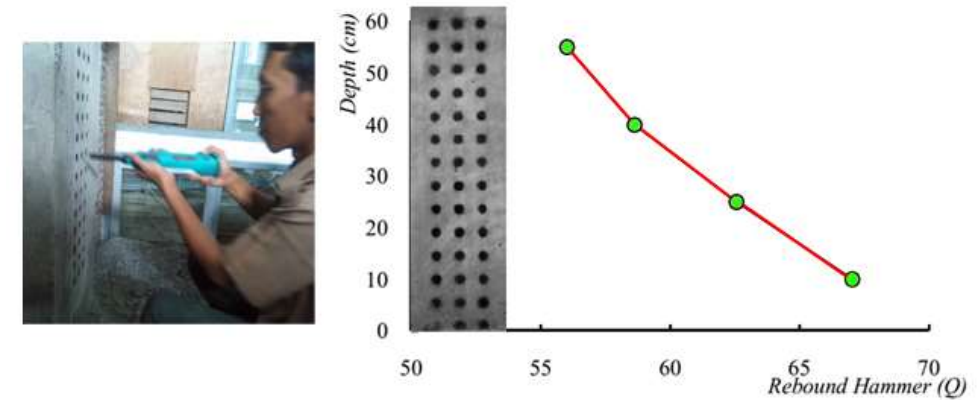
Bamboo



Iron-Copper FGM

FGC = FUNCTIONALLY GRADED CONCRETE

- Studies on the FGC are very limited.
- Attempts to manufacture an FGC material face one challenging difficulty. In a laminated or composited material, the stress concentrations will occur and degrade the quality of the FGC material.
- Numerically, a study on the effects of two concrete strengths gradation of FGC specimens has been reported that the ultimate strengths of the FGC were limited by the lowest concrete strength of the FGC and their rigidities are close to the highest compressive strength of the FGC mixture.

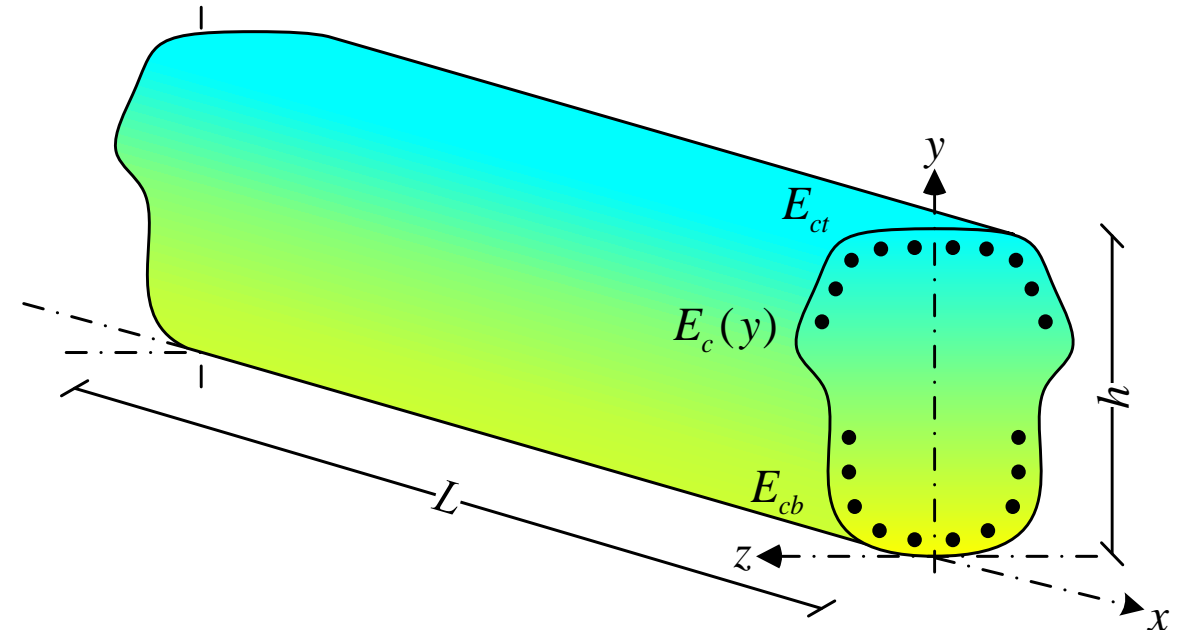


WHAT IS THE PROBLEM?

- FGC has **not been implemented** widely in construction projects.
- One major problem in implementing FGC is that there are **no building codes available** for designing FGC elements in buildings.
- In this paper, we show a design method of steel **Reinforced Functionally Graded Concrete (RFGC)** beam subjected to a bending moment.
- A study on price comparison is conducted to highlight the **economic feasibility** of the RFGC.

FGM CONCEPT TO RC BEAM

- By grading two different types of the concrete strengths throughout the thickness of the cross-section of a beam, it is possible to reduce the unnecessary concrete strength in the tension zone and to increase the strength in the compressive zone.
- Optimally, this can potentially reduce the material prices.
- Similar ideas could lead to the enhancement of a wide range of other building components.



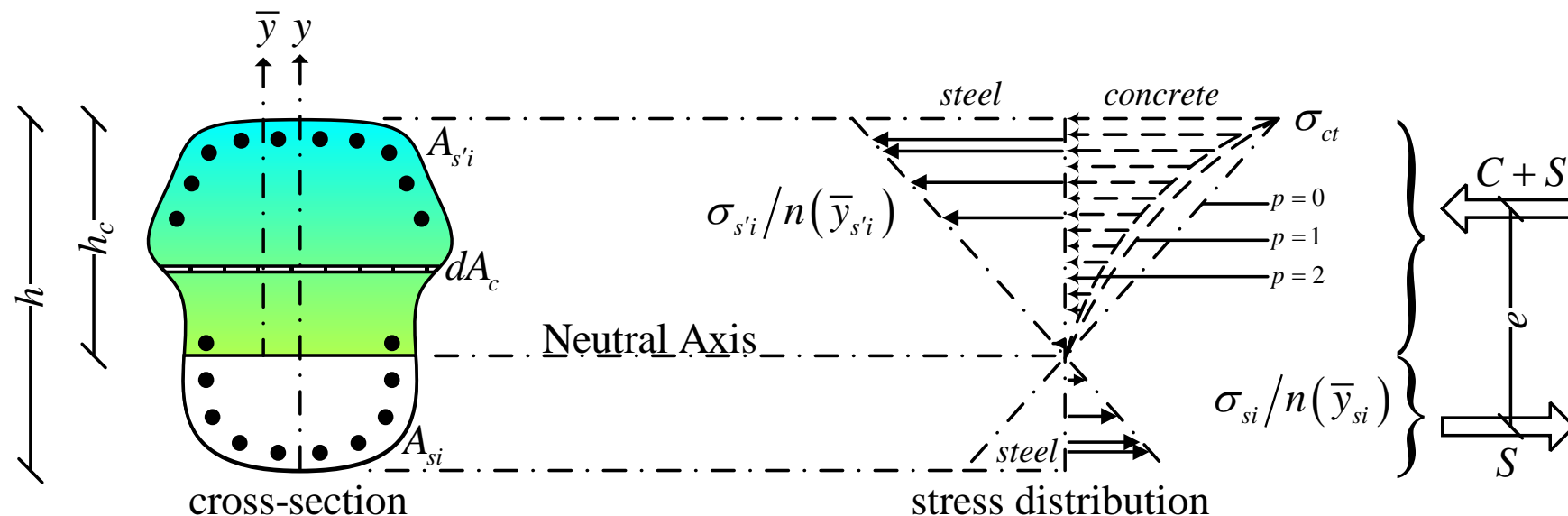
$$E_c(y) = E_{cb} \left[1 + \frac{E_{ct} - E_{cb}}{E_{cb}} \left(\frac{y}{h} \right)^p \right]$$

ASD METHOD OF RFGC

- The **allowable stress design** (ASD) code standard for designing an RC beam subjected to **bending moment** has been used for many years.
- In the ASD, the following assumptions are made:
 - In the calculation of stresses at the FGC and steel bars at a section, the **tensile** strength of the assumed **cracked concrete** part below the natural axis is neglected.
 - The dimension of the **length** of the beam is **relatively long** compared to the maximum dimension of the cross-section. Hence, the **section remains plane** and perpendicular to the neutral axis of the beam after the deformation.
 - The material properties of steel and concrete are **linear elastic**.

ASD METHOD OF RFGC

- The **kinematic** of the RFGC beam cross-section under a bending moment is illustrated in the figure below. The compression forces consist of the uncracked concrete area and steel bar in compression, while the tensile force is only resisted by the steel bar in the cracked concrete area.
- The **design** of the beam cross-section is **iterated** by the **calculation** of balancing moment strengths controlled by the concrete and steel.



ASD METHOD OF RFGC

$$n(y) = \frac{E_s}{E_c(y)}$$

$$S = C + S'$$

$$S = \frac{\sigma_{ct}}{h_c} \sum_{i=1}^{ns} n(\bar{y}_{si}) \bar{y}_{si} A_{si}$$

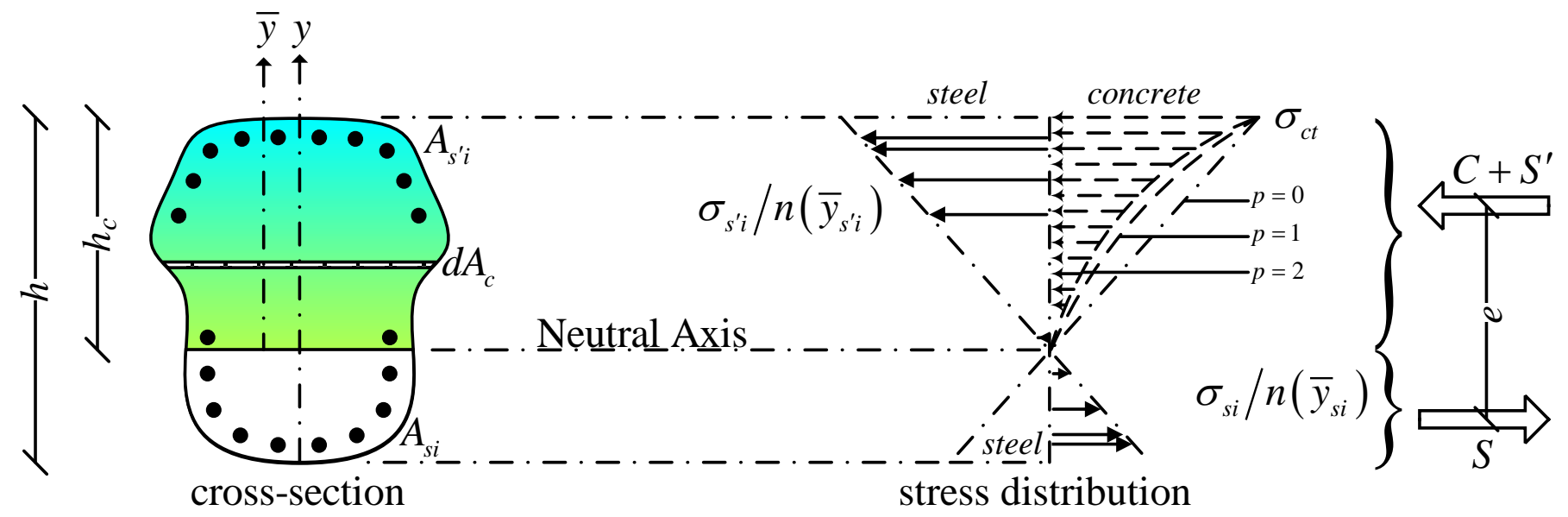
$$S' = \frac{\sigma_{ct}}{h_c} \sum_{i=1}^{ns'} n(\bar{y}_{s'i}) \bar{y}_{s'i} A_{s'i}$$

$$C = \frac{\sigma_{ct}}{h_c} \int_{A_c} \bar{y} dA_c$$

$$\int_{A_c} \bar{y} dA_c + \sum_{i=1}^{ns'} n(\bar{y}_{s'i}) \bar{y}_{s'i} A_{s'i} - \sum_{i=1}^{ns} n(\bar{y}_{si}) \bar{y}_{si} A_{si} = 0$$

$$I = \int_0^{y_c} \bar{y}^2 dA_c + \sum_{i=1}^{ns'} n(\bar{y}_{s'i}) \bar{y}_{s'i}^2 A_{s'i} - \sum_{i=1}^{ns} n(\bar{y}_{si}) \bar{y}_{si}^2 A_{si} = 0$$

$$e = \frac{I}{\int_{A_c} \bar{y} dA_c + \sum_{i=1}^{ns'} n(\bar{y}_{s'i}) \bar{y}_{s'i} A_{s'i}} = \frac{I}{\sum_{i=1}^{ns} n(\bar{y}_{si}) \bar{y}_{si} A_{si}}$$



$$\sigma_{ct} = \frac{M h_c}{I}$$

$$\sigma_{si} = n(\bar{y}_{si}) \frac{\sigma_{ct} \bar{y}_{si}}{h_c}$$

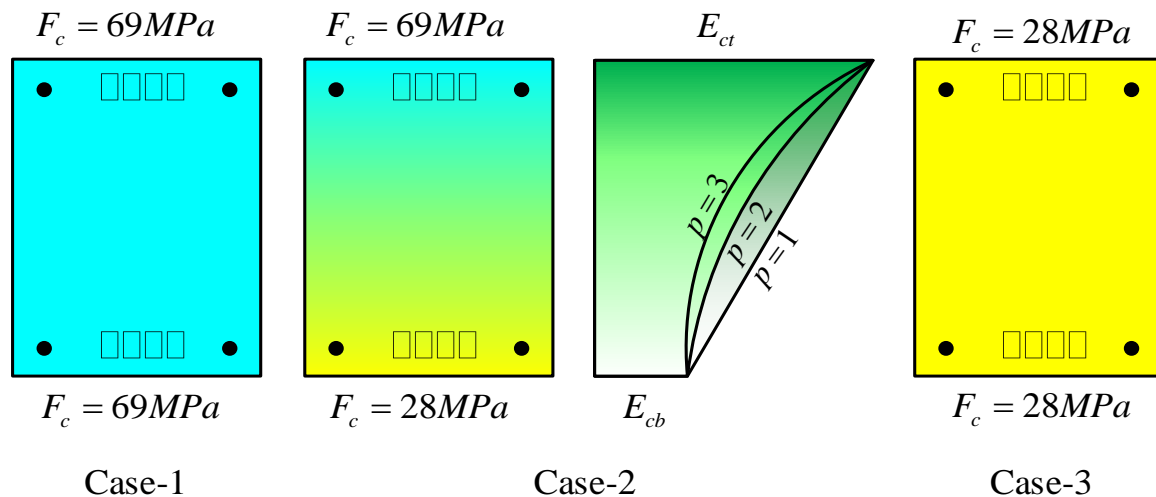
$$\sigma_{s'i} = n(\bar{y}_{s'i}) \frac{\sigma_{ct} \bar{y}_{s'i}}{h_c}$$

$$M_{sr} = \frac{\bar{\sigma}_s I}{n(\bar{y}_{sb}) \bar{y}_{sb}}$$

$$M_{cr} = \frac{\bar{\sigma}_c I}{h_c}$$

DESIGN OF RECTANGULAR RFGC BEAMS

- Two homogeneous RC beams of 69 MPa and 28 MPa concrete compressive strengths (Case-1 and 3) are considered. Case-2 shows an RFGC beam cross-section of functionally graded concrete compressive strengths which vary from 28 MPa at the bottom fiber and 69 MPa at the top fiber of the beam cross-section.
- In this study, the graded function is selected to follow the degree of polynomial order of $p = 1$ (linear), 2 (quadratic) and 3 (cubic) variation of cases. In practice, the quadratic or cubic functions are more likely to be found in real concrete structures.



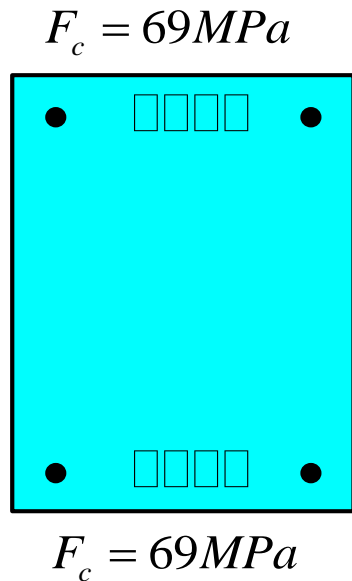
$$E_c = 2.10 \times 10^4 \times \left(\frac{\gamma_c}{23}\right)^{3/2} \times \left(\frac{F_c}{20}\right)^{1/2} \quad (F_c \leq 36 \text{ N/mm}^2)$$

$$E_c = 3.35 \times 10^4 \times \left(\frac{\gamma_c}{24}\right)^2 \times \left(\frac{F_c}{60}\right)^{1/3} \quad (F_c > 36 \text{ N/mm}^2)$$

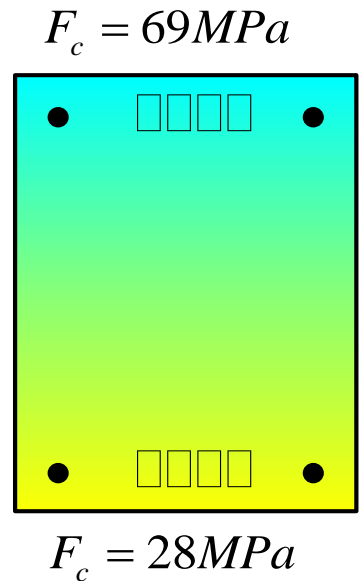
$$\gamma_c(y) = \gamma_{cb} \left[1 + \frac{\gamma_{ct} - \gamma_{cb}}{\gamma_{cb}} \left(\frac{y}{h}\right)^p \right]$$

DESIGN OF RECTANGULAR RFGC BEAMS

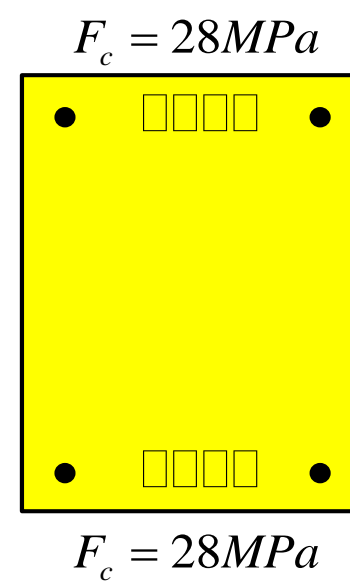
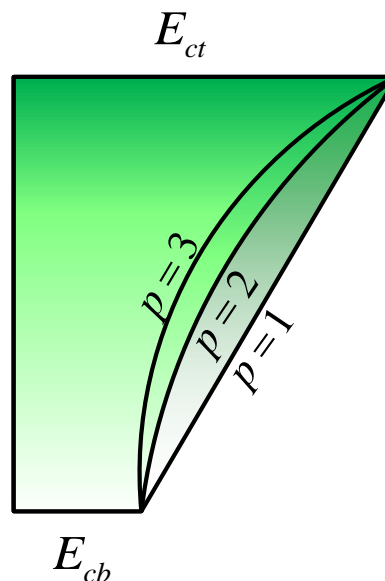
- The volume fraction of concrete strength compositions can be calculated.



Case-1



Case-2



Case-3

$$V = \int_0^h \left\{ \gamma_{cb} \left[1 + \frac{\gamma_{ct} - \gamma_{cb}}{\gamma_{cb}} \left(\frac{y}{h} \right)^p \right] b(y) \right\} dy$$

for $p = 0 \rightarrow V_{ct} = V_{cb} = V$

for $p = 1 \rightarrow V_{ct} + V_{cb} = 2V$

for $p = 2 \rightarrow V_{ct} + 2V_{cb} = 3V$

for $p = 3 \rightarrow V_{ct} + 3V_{cb} = 4V$

PRICE MATERIAL CALCULATION

- Price Material and volume of concrete and steel of all cases.

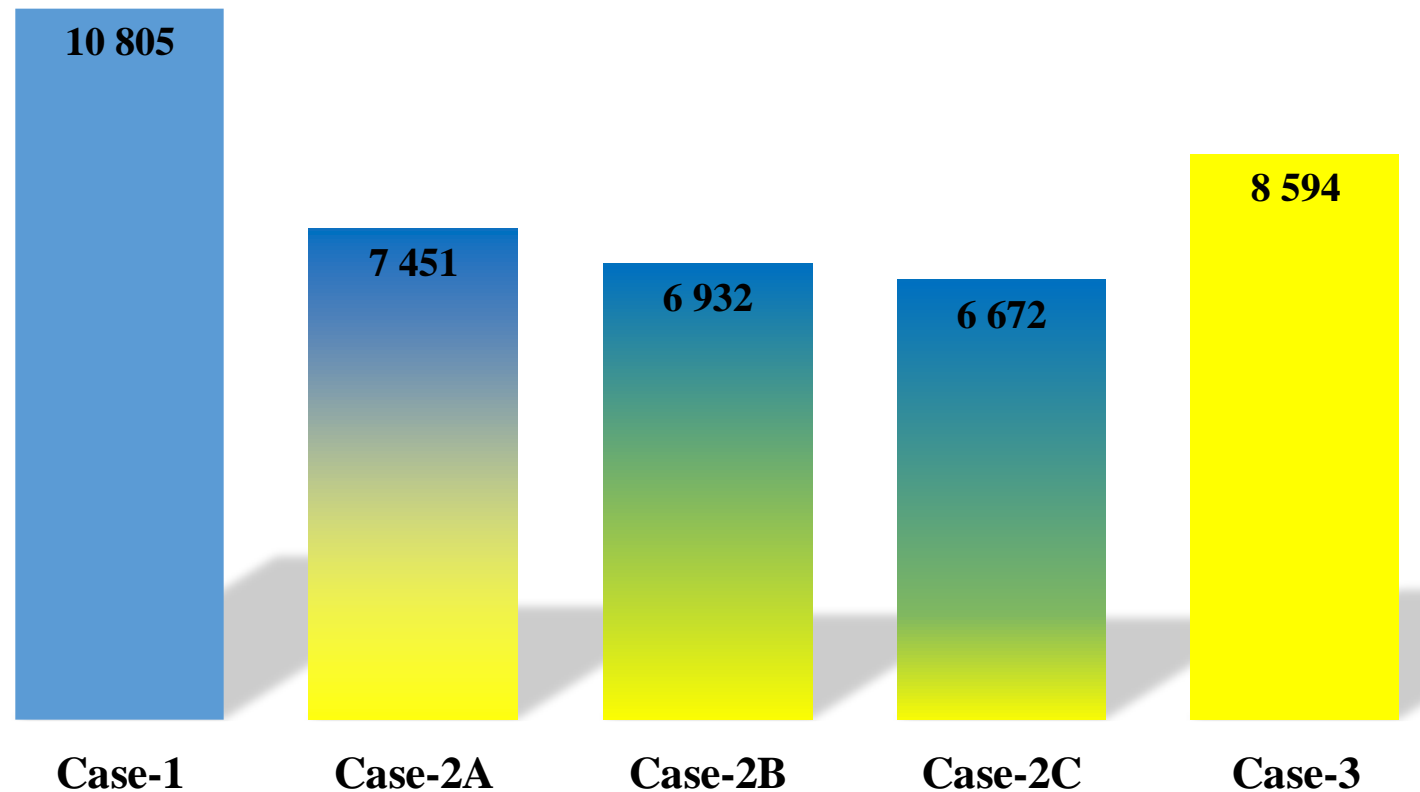
Strength F_c (MPa)	YEN/m ³
28	17 400
69	34 700

Steel SD-345 (YEN/Ton)
67 000

Case	$b \times h$ (mm \times mm)	Concrete volume (m ³) per unit length		p	Steel bars (top) (bottom)	Total price per meter length (Yen)
		$F_c = 28$ MPa	$F_c = 69$ MPa			
1	320 \times 600	0.0000	0.1920	0	2 \times 3 \times ϕ 29 2 \times 3 \times ϕ 29	10 805 (+25.7%)
2A	300 \times 600	0.0900	0.0900	1	1 \times 2 \times ϕ 29 2 \times 3 \times ϕ 29	7 451 (-13.3%)
2B	300 \times 600	0.1200	0.0600	2	1 \times 2 \times ϕ 29 2 \times 3 \times ϕ 29	6 932 (-19.3%)
2C	300 \times 600	0.1350	0.0450	3	1 \times 2 \times ϕ 29 2 \times 3 \times ϕ 29	6 672 (-22.4%)
3	450 \times 770	0.3465	0.0000	0	1 \times 5 \times ϕ 25 1 \times 5 \times ϕ 25	8 594

RESULTS AND CONCLUSIONS

- Price of RFGC beam per unit length in Japanese Yen.



RESULTS AND CONCLUSIONS

- The 3rd degree of polynomial assumption was found to be the **most effective combination** for FGC material. Therefore, no need to produce the linear ($p = 1$) FGC which is, indeed, more difficult to manufacture.
- By using the RFGC (Case-2), the **weight** of the **normal RC** (Case-3) can be reduced by **41.8% less**, and thereby, **lightweight structure** and **more spaces** can be gained in designing **high-rise building**.
- Overall, the RFGC beams are **more economical** than both the normal and high strength RC beams.

